Using mathematical induction, prove that $2^3 + 4^3 + \cdots + (2n)^3 = 2n^2(n+1)^2$ for all positive integers n. **SCORE:** _____ / 25 PTS

BASIS STEP: $2^3 = 8$ (2) $2(1)^2(1+1)^2 = 2(1)(4) = 8$ (2)

INDUCTIVE STEP: ASSUME $2^3 + 4^3 + \cdots + (2k)^3 = 2k^2(k+1)^2$ FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k \geq 1$ (3) (2)

(3) $2^3 + 4^3 + \cdots + (2k)^3 + (2(k+1))^3$

(3) $= 2k^2(k+1)^2 + (2(k+1))^3$

$= 2(k+1)^2[k^2 + 4(k+1)]$

$= 2(k+1)^2(k^2 + 4k + 4)$ (5)

(3) $= 2(k+1)^2(k+2)^2$

BY MI, $2^3 + 4^3 + \cdots + (2n)^3 = 2n^2(n+1)^2$ FOR ALL INTEGERS $n \geq 1$ (2)

A cat jumps from a window of a building, at an angle of 53.13° above the horizontal, with an initial speed of 7.5 feet per second, and lands safely on the ground 9 feet away from the base of the building. Let h be the height of the window above ground level.

SCORE: _____ / 25 PTS

NOTE: $\sin 53.13^\circ = \frac{4}{5}$ **and** $\cos 53.13^\circ = \frac{3}{5}$

[a] Write parametric equations for the cat's position.

$x = (7.5 \cos 53.13^\circ)t = 4.5t$ (4)

$y = h + (7.5 \sin 53.13^\circ)t - 16t^2 = h + 6t - 16t^2$ (4)

[b] How high off the ground is the window ?

$x = 4.5t = 9$ (3)

$t = 2$ (3)

$y = h + 6(2) - 16(2)^2 = 0$ (3) (3) GROUND LEVEL

$h = 52$ FEET (2)

(3)

For the month of October 2013, MJ's and NJ's water bills were each $23. Over the next two years, MJ's monthly bill has increased 25 cents each month, while NJ's monthly bill has increased 1% each month. Whose total water bills over the last two years has been higher, and by how much ? **SCORE: _____ / 20 PTS**

(4)

MJ: $S_{24} = \frac{24}{2}(2(23) + (0.25)(24-1)) = 621$

NJ: $S_{24} = \frac{23(1.01^{24}-1)}{1.01-1} = 620.39$

MJ'S BILLS TOTALLED 61 CENTS MORE THAN NJ'S

(2) POINTS EXCEPT AS NOTED

Use sigma notation to write the series $-\frac{231}{37}+\frac{217}{54}-\frac{203}{71}+\frac{189}{88}-\cdots+\frac{21}{292}$.

SCORE: ____ / 15 PTS

$-14 \quad -14 \quad -14$

$+17 \quad +17 \quad +17$

$$\sum_{n=1}^{16} (-1)^n \frac{231-14(n-1)}{37+17(n-1)}$$

(2) (2) (2) ($3\frac{1}{2}$) ($3\frac{1}{2}$)

$37+17(n-1)=292$ (2)

$17(n-1)=255$

$n-1=15$

$n=16$

(-2) IF INDEX DOESN'T MATCH INSIDE OF SUMMATION

Eliminate the parameter to find a rectangular equation corresponding to the parametric equations $\begin{aligned} x &= 4 + 5\sec t \\ y &= 3 + 7\tan t \end{aligned}$. **SCORE: _____ / 15 PTS**

For your final answer, write y as a **simplified** function of x.

$\sec t = \frac{x-4}{5}$ (4)

$\tan t = \frac{y-3}{7}$ (4)

$\sec^2 t - \tan^2 t = 1$

$\left(\frac{x-4}{5}\right)^2 - \left(\frac{y-3}{7}\right)^2 = 1$ (7)

Simplify $\dfrac{(3n-2)!}{(3n+1)!}$.

SCORE: _____ / 10 PTS

$$\frac{(3n-2)!}{(3n+1)(3n)(3n-1)(3n-2)!} = \frac{1}{3n(3n+1)(3n-1)}$$

(6) (4)

Find the value of $\sum_{n=2}^{\infty} 500(0.8)^{2n-1}$. **HINT: Write out the first few terms first.**

SCORE: _____ / 15 PTS

$500(0.8)^3 + 500(0.8)^5 + 500(0.8)^7 + \cdots$

$* (0.8)^2$ $* (0.8)^2$

(4)

$$S = \frac{500(0.8)^3}{1 - (0.8)^2} = 711.\overline{1}$$

(3) (4) (4)

Consider the expression $(9x^8 - 7x^5)^{27}$.

SCORE: ____ / 25 PTS

[a] Write the first 3 terms of the expansion of the expression. Simplify all exponents.

Your answer may use multiplication and exponents, but NOT division, ! nor ${}_nC_r$ (or equivalent) notation.

$$\binom{27}{0}(9x^8)^{27} + \binom{27}{1}(9x^8)^{26}(-7x^5) + \binom{27}{2}(9x^8)^{25}(-7x^5)^2$$

$$= \frac{27!}{0!\,27!}\,9^{27}x^{216} + \frac{27!}{1!\,26!}\,9^{26}(-7)x^{213} + \frac{27!}{2!\,25!}\,9^{25}(-7)^2x^{210}$$

$$= 9^{27}x^{216} - 27\cdot 9^{26}\cdot 7x^{213} + \frac{27\cdot 26}{2}\,9^{25}7^2x^{210}$$

(1) POINT EXCEPT AS NOTED

$$= 9^{27}x^{216} - 27\cdot 9^{26}\cdot 7x^{213} + 27\cdot 13\cdot 9^{25}\cdot 7^2x^{210}$$ (2)

[b] Find the coefficient of x^{93} in the expansion.

Your answer may use multiplication, division, exponents and !, but NOT ${}_nC_r$ (or equivalent) notation.

$$\binom{27}{r}(9x^8)^{27-r}(-7x^5)^r$$

$$= \frac{27!}{r!(27-r)!}\,9^{27-r}(-7)^r x^{216-3r}$$

(6) $216-3r=93$

$-3r=-123$

(2) $r=41>27$

(2) NO SUCH TERM (x^{93})

BONUS (5) COEFFICIENT = 0

OR

(10) LOWEST POWER OF x IS $(x^5)^{27} = x^{135}$